Side Holes in Cylinders. The present discussion is concerned only with the stress concentration induced in a cylinder under internal pressure by the presence of circular side holes and elliptic side holes, in which the major axis of the ellipse is perpendicular to the longitudinal axis of the cylinder (Figure 1).

In trying to arrive at theoretical values of stress concentration factors at the bore–side hole interface, the first thought is to treat the cylinder bore like a plate with a hole (Equation 1). When the hole also has pressure in it, one would intuitively expect K factors (stress concentration factors) in excess of 3 and possibly reaching values as high as 5 or 6. Experimental data and theoretical analysis do not support this type of reasoning. In the following analysis the plate theory above is used; first, however, it is necessary to determine how to apply plate theory in the case of a circular cylinder. The distribution of stress in a cylinder containing side holes, where both cylinder bore and side holes are subjected to internal pressure, is complex and thus it is necessary to reduce the situation to a simpler one that is amenable to exact analysis. This is accomplished by noting that the addition of a hydrostatic stress on a system does not alter its fundamental behavior.

Figure 3 shows a closed-end cylinder containing an elliptic side hole subjected to internal pressure; to this picture hydrostatic tension (Figure 4) equal in magnitude to the internal pressure is added. Superposition then gives the situation shown in Figure 5, which is amenable to exact analysis by Lamé's equations for a cylinder subjected to external pressure. Thus, for the cylinder in Figure 5

$$\sigma_h = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right) \tag{6}$$

$$\sigma_r = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right) \tag{7}$$

$$\sigma_z = \frac{p_o r_o^2}{r_o^2 - r_i^2} \tag{8}$$

where

σ_h, σ_r, σ_z = circumferential, radial, and longitudinal stresses in cylinder
p_o = external pressure
r_i, r_o = inside and outside radii of cylinder
r = variable radius in cylinder

The location of maximum stress concentration is at A on the bore surface, Figure 5; therefore, in Equations 6 and 7, $r = r_i$ and

$$\sigma_{h(\max)} = \frac{2p_o R^2}{R^2 - 1} \tag{9}$$

$$\sigma_r = 0 \tag{10}$$

$$\sigma_z = \frac{p_o R^2}{R^2 - 1} \tag{11}$$

where $R = r_o/r_i$

Elastic plate analysis can now be applied to the cylinder problem. The hoop stress generated at the bore of the cylinder (Equation 9) is interpreted as the unit stress, S_x, in Equation 2; thus

$$\sigma_x = \frac{2p_o R^2}{R^2 - 1}\left(1 + 2\frac{b}{a}\right) \tag{12}$$

Because the cylinder has closed ends, a longitudinal stress is generated (Equation 11) which is interpreted as unit stress S_y in Equation 4, so that

$$\sigma_x = \frac{p_o R^2}{R^2 - 1} \tag{13}$$

Stresses σ_x given by Equations 12 and 13 are both generated at location A, Figure 5, and are additive; thus, the total effective hoop stress in the cylinder at A is as follows:

$$(\sigma_h)_A = \frac{2p_o R^2}{R^2 - 1}\left(1 + 2\frac{b}{a}\right) - \frac{p_o R^2}{R^2 - 1} \tag{14}$$

The normal hoop stress at A is given in Equation 9; therefore, at this location the stress concentration factor, K, is

$$K = \frac{\dfrac{p_o R^2}{R^2 - 1}(1 + 4\,b/a)}{\dfrac{2p_o R^2}{R^2 - 1}} = \frac{1 + 4\,b/a}{2} \tag{15}$$

Use of Equation 15 now permits the calculation of specific stress-concentration factors depending on the geometry of the side holes. If the side hole is small and circular, $a = b$ in Equation 15 and $K = 2.5$. For an elliptic hole of geometry $a/b = 2$, $K = 1.50$. These K values indicate the influence of the longitudinal stress in a cylinder on stress concentration at side holes—for example, with no longitudinal stress and a circular side hole K would be 3.0, the same as in a plate under tension—the presence of the longitudinal stress decreases this value to 2.5, or a decrease of 16.7%.

The K factor is determined after the application of hydrostatic tension to the cylinder (Figure 5). Therefore, in using the above theory to solve practical problems, one must keep this fact in mind and make calculations accordingly. For example, suppose the cylinder has open rather than closed ends; in this case, the effect of superposition of hydrostatic tension would give the result in Figure 5 minus the tensile force distributed over the bore area and the resultant longitudinal stress would be p_o rather than the stress expressed by Equation 11. Thus, in the paragraph above zero longitudinal stress could result only by the application of a compression force to the end of the cylinder, which would be canceled by the hydrostatic tension. For an open-end cylinder the K factor for a small circular hole would be not 3.0 but $3.0 - \left(\frac{R^2 - 1}{2R^2}\right)$; in other words, for an open-end cylinder the K factor depends on the wall ratio, R.

The foregoing portion of the analysis

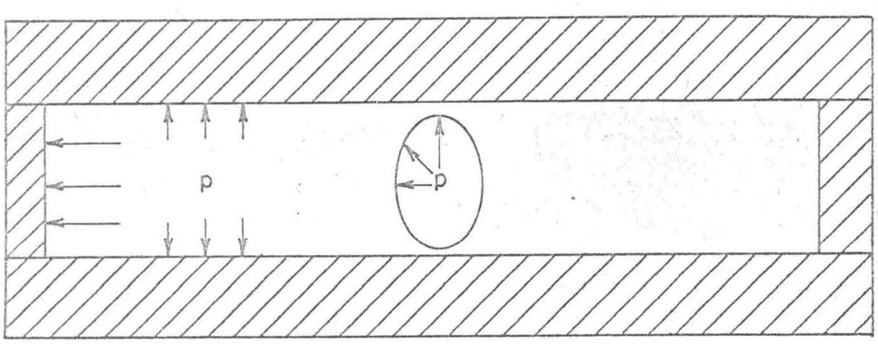

Figure 3. Closed-end cylinder with side hole subjected to internal pressure

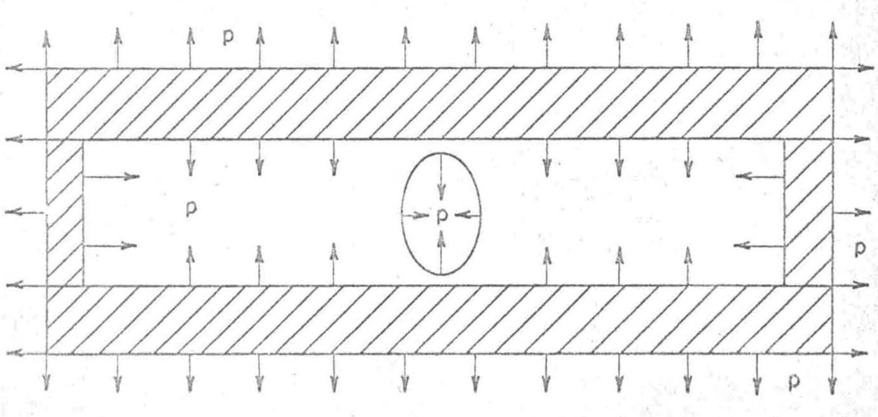

Figure 4. Cylinder subjected to hydrostatic tension